Compression of femtosecond light pulses by one-dimensional photonic crystals with two-component relaxing nonlinearity

R. A. Vlasov* and A. G. Smirnov†

Stepanov Institute of Physics of the National Academy of Sciences of Belarus, 70 F. Scaryna Avenue, 220072 Minsk, Belarus

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The compression of ultrashort light pulses in a one-dimensional nonlinear photonic crystal is investigated for light frequencies lying outside the forbidden gap of a periodic structure taking into account the delayed nonlinear response of a medium. It is shown that the relatively slow-responding defocusing nonlinearity can suppress the distortion of Bragg solitons caused by the relaxation of the fast self-focusing component and improve the compression efficiency. Stable optical quasisoliton propagation of light pulses and their compression are numerically confirmed for both band gap composite materials and photonic crystals with alternating layers of different types of nonlinearity.

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I. INTRODUCTION

The application of periodic structures for the compression of light pulses at visible and infrared frequencies has a number of advantages in comparison with traditional techniques. The dispersion of such structures originating from a spatial ordering of a medium is several orders of magnitude greater than the inherent dispersion of dielectric materials at nonresonant frequencies. As a result, the compression length is significantly reduced. Moreover, one can fabricate a periodic sample in a desirable manner to provide appropriate dispersion properties of a crystal in a necessary spectral range. The possibility of compression of picosecond light pulses in periodic media was theoretically predicted by Winful $[1]$ and experimentally realized by Eggleton and co-workers [2] on the base of weakly modulated distributed feedback waveguides. Later the authors of Ref. $[3]$ suggested pulse compression methods utilizing the so-called photonic crystals or band gap (PBG) structures, i.e., one-dimensional or multidimensional periodic formations with the large depth of modulation of refractive index. It was shown that such either natural or artificial crystals are the ultimate means for the controlled compression of powerful laser pulses up to the duration of several optical cycles on the submillimeter spatial scale. However, the authors of Ref. $[3]$ neglected the relaxation of the nonlinear medium response, whereas even the least inertial medium, i.e., having the shortest relaxation time, may not be considered as the instantaneously responding one at the femtosecond pulse duration chosen. This ignorance of the relaxation process seems to be unsound. For example, in homogeneous optical fibers the prompt, but yet finite, response of the cubic nonlinearity, that is usually treated in terms of the intropulse Raman scattering, leads to the frequency self-shift of light pulses and their shape distortion $|4|$. Similar effects are surely inevitable in periodic structures, and if so, they do impose severe limitations on the

realization of the light pulse compression in PBG crystals. On the other hand, the recent investigation on soliton dynamics in two-component composite homogeneous materials $[5]$ has demonstrated the feasibility of soliton distortion compensation under the joint action of relatively fast-responding self-focusing and slow-responding defocusing nonlinear mechanisms. The pulse duration is supposed to be intermediate relative to the short and long relaxation times of both nonlinearity components. Then, under a certain relationship between parameters of a medium and light pulse, the substantial mutual compensation of distortion is achieved.

In the present paper, on the basis of finite-difference timedomain numerical simulation we investigate the influence of relaxation processes on the compression of ultrashort light pulses in a one-dimensional (1D) nonlinear band gap structure. The idea of a two-time-scale nonlinear response can be expected to be effective here, too. Pulse frequencies are assumed to lie in the close vicinity to, but definitely outside the forbidden gap. Both band gap composite materials and photonic crystals with alternating layers of different types of nonlinearity are investigated for the case when the delay of relatively fast nonlinear response is comparable with the pulse duration. To our knowledge, systematic studies on this subject are absent in the literature as yet. In this connection, the problem in point appears to be of theoretical and practical interest.

II. BASIC CONCEPTS AND ASSUMPTIONS

Let us consider the formation of solitonlike pulses within a finite 1D periodic structure fabricated as a stack of layers with two different sorts of dielectric materials. The linear part of refractive index of such a crystal $n_0(z)$, varies along the *Z* axis and takes on n_1 or n_2 for each sort of layers. For the so-called photonic crystals under consideration the modulation depth $n_1/n_2 > 1.3$ and the size of the elementary lattice cell $d=d_1+d_2$, where d_1 , d_2 are layers thicknesses, lies in the optical range. In the linear regime such a stack operates as a Bragg reflector for light frequencies being within the forbidden gap and exhibits strong dispersion properties in the near-gap spectral range. In the case of a cubic nonlinear material embedded inside a crystal one obtains a

^{*}FAX: 375-17-2840879. Electronic address: evp@dragon.basnet.by

[†] FAX: 375-17-2840879. Electronic address: andr@dragon.basnet.by

compression effect arising from the interplay between the group velocity dispersion (GVD) of a grating and self-phase modulation of a light pulse $[1]$. At frequencies lying far from the forbidden gap the coupling between direct and backward waves is not very strong and an incident pulse passes mainly through a crystal without significant reflection. Therefore it is possible to describe the light pulse compression in gap structures in the same manner as in a homogeneous isotropic dispersive nonlinear medium $[3,6]$. Namely, one introduces a characteristic length of the pulse dispersion broadening L_D and characteristic length of the nonlinear self-phase modulation L_N to evaluate the balance between two principal mechanisms responsible for the soliton formation. In terms of the second order dispersion for the Gaussian shape of an incident pulse

$$
E(t)|_{z=0} = E_0 \exp\left[-\frac{t^2}{2\tau_0^2}\right] \exp i\omega_0 t, \qquad (2.1)
$$

these parameters are expressed in the following form $[7]$:

$$
L_D = \frac{\tau_0^2}{|k_2|}; \quad L_N = \tau_0(|k_2|\omega_0 \tilde{n}_2 I_0/c)^{-1/2}, \tag{2.2}
$$

where τ_0 is the pulse duration, ω_0 is the light career frequency, I_0 is the pulse intensity, \tilde{n}_2 is the nonlinear cubic coefficient of the medium, *c* is the speed of light in vacuum, and $k_2 = d^2k/d\omega^2$ is the group velocity dispersion of a crystal. Imposing the condition of Bragg soliton formation $[7]$, $L_D=L_N$, one estimates the level of intensity I_c needed

$$
I_c = \frac{c}{\omega_0 \tilde{n}_2 L_D}.\tag{2.3}
$$

In general, the pulse compression takes place, if the condition $\tilde{n}_2 k_2 < 0$ is satisfied [3,7]. Therefore in the particular case of a self-focusing nonlinear medium with \tilde{n}_2 > 0 one attains the compression effect at light frequencies lying within a spectral band where the GVD is negative $(Fig. 1)$. For typical parameters of 1D gap structures the quantity k_2 varies by more than several orders in the area of the anomalous or negative dispersion and the GVD is extremely high in the vicinity of the gap edge. The evaluation shows that $|k_2|$ $\sim 10^2$ fs²/ μ m corresponds to the very short compression length $L \sim 40-80d$ for the pulse duration $\tau \sim 10-30$ fs. However, in this case the compression realization requires the light intensity $\sim 10^{14}$ W/cm². Such a magnitude lies near the optical damage threshold of dielectric lossless materials for the pulse duration \sim 10 fs [7].

Since the problem under investigation is characterized by the ultrashort duration $\tau \sim 10-10^2T$ of an incident pulse, *T* $=2\pi/\omega_0$, and the inherent inhomogeneity of a medium is comparable with the light wavelength inside a crystal, for the interaction dynamics to be treated adequately we numerically solve the original Maxwell equations. In the one-dimensional formulation they are written as

$$
\frac{\partial E}{\partial z} = \frac{1}{c} \frac{\partial H}{\partial t},\tag{2.4a}
$$

FIG. 1. Compression of 30 fs pulse at $\lambda = 1.064$ μ m in PBG structure; $\tilde{n}_2 I_0 = 0.005$, $|k_2| = 16$ fs²/ μ m; the output signal in the linear regime is also depicted for comparison. The parameters of the photonic crystal are $n_1=1.41$, $d_1=0.22$ μ m, $n_2=1.00$, d_2 $=0.31$ μ m, $L=110d$. The insert depicts the crystal reflectivity spectrum (solid curve) and spectrum of an incident pulse (dashed curve).

$$
\frac{\partial H}{\partial z} = \frac{1}{c} \frac{\partial D}{\partial t},\tag{2.4b}
$$

$$
D(z,t) = n^2(z,t)E(z,t),
$$
 (2.4c)

where *E*,*H* are the electric and magnetic fields, respectively; $n(z,t)$ is the nonlinear refractive index of a structure. We deal with the Debye model of the cubic nonlinearity $[7]$ describing the delayed nonlinear response of a dielectric material at nonresonant frequencies with relaxation time t_0 :

$$
n(z,t) = n_0(z) + \delta n(|E(z,t)|^2),
$$
 (2.5a)

$$
t_0 \frac{d\delta n}{dt} + \delta n = \tilde{n}_2(z)|E(z,t)|^2.
$$
 (2.5b)

A finite-differences time-domain technique is applied for the solution of Eqs. (2.4) , in particular the Lax-Wendroff algorithm. In addition, a specific transparent boundary condition obtained in Ref. $[8]$ is imposed, which enables us to perform simulating a time-dependent pulse source at the input boundary without sharing additional grid cells for splitting counter-propagating signals and excluding nonphysical perturbations. The boundary conditions mentioned also provide the stability of computing the pulse dynamics in a medium with the defocusing nonlinearity \tilde{n}_2 <0, when the local intensity-dependent speed of light $c/n(|E|)$ may be lower than the average one throughout the crystal. Our numerical experiments are intended for the joint solution of Eqs. (2.4) and master Eqs. (2.5) .

III. LIGHT COMPRESSION IN COMPOSITE BAND GAP MATERIALS

Dealing with the compression of ultrashort pulses of ten optical cycles ($\tau_0 \sim 10-30$ fs for $\lambda = 1.064$ μ m) in photonic crystals, one runs into severe difficulties concerning the choice of the fast-responding nonlinear materials. The electronic mechanism of cubic response in solids as the fastest one is mostly preferable, but even in this case the relaxation time t_0 is comparable with the pulse duration. For example, in quartz glasses the value of t_0 is estimated to be \sim 3–6 fs @7#. As follows from numerical simulations, this makes it almost impossible to attain the compression effect at any level of input intensity for the ultrashort pulse duration. Figure 2 depicts the results of compression of the Gaussian pulse at τ =30 fs after passing through *N*=110 cells of the photonic crystal with delayed-response self-focusing nonlinear material concentrated in the layers of lower refractive index. As is seen from the picture, the compression deteriorates at $t_0 / \tau_0 \sim 0.2$.

The situation becomes better when the photonic crystal contains a composite nonlinear material with the twocomponent nonlinearity providing the compensation of destructive relaxation effects $[5]$. Under this condition the slow-responding defocusing nonlinearity counteracts to the fast-responding self-focusing one, so that distortions contributed by the pertinent relaxation processes turn out to be opposite. Thereby the compensation mentioned is achieved dynamically. The nonlinear part of the refractive index of a composite material may be written as a sum of two nonlinear terms

$$
\delta n = \delta n^+ + \delta n^-, \tag{3.1}
$$

where δn^+ and δn^- obey the differential equations

$$
t_0^+ \frac{d \delta n^+}{dt} + \delta n^+ = \tilde{n}_2^+(z) |E(z, t)|^2, \tag{3.2a}
$$

$$
t_0^{-} \frac{d \delta n^{-}}{dt} + \delta n^{-} = \tilde{n}_2^{-}(z) |E(z, t)|^2.
$$
 (3.2b)

The coefficients $n_2^+ > 0$ and $n_2^- < 0$ are related to the selffocusing and defocusing nonlinearities, respectively. In the present model we assume that both nonlinear mechanisms act simultaneously and locally in space.

Let us introduce a set of scaling parameters for the consideration below:

$$
\varepsilon_1 = \frac{t_0^+}{\tau_0}, \ \varepsilon_2 = \frac{\tau_0}{t_0^-}, \ \varepsilon_3 = \frac{\tilde{n}_2^-}{\tilde{n}_2^+}, \ \xi = \varepsilon_1 \varepsilon_2 \varepsilon_3.
$$
 (3.3)

Since it is well known that the less inertial material possesses the lower nonlinear coefficient $\tilde{n}_2^-\sim t_0$ [7], in general, the parameter ξ should be given a value being comparable with unity. The theoretical analysis of the solution of the Schroedinger equation for the soliton propagation in a homogeneous dispersive two-component medium [5] predicts almost the full compensation of soliton distortion at much lower values of $\xi \sim 0.0031$ under condition when the inequality

FIG. 2. Transmitted signal for the light pulse compression in PBG at various values of the relaxation time t_0 of the fast nonlinear response: (a) $\tilde{n}_2 I_0 = 0.005$, (b) $\tilde{n}_2 I_0 = 0.007$. Normalized time is calculated as $t' = (t - t_p)/\tau_0$, normalized intensity—as $I/|E_0|^2$, incident envelope is depicted relatively shifted at $t_p = (n_1 d_1)$ $+n_2d_2$)*N/c*. Radiation and crystal parameters are the same as in Fig. 1.

$$
t_0^+ \ll \tau_0 \ll t_0^- \tag{3.4}
$$

holds. While compressing the light pulses of $\tau_0 \sim 30$ fs in the dielectric PBG structures, the quantity $\varepsilon_1 \sim 0.1-0.2$ may not be considered as the small one anyway. Thus in our numerical experiments we study the possibility of compensation of Bragg soliton distortion in the composite photonic crystals for

$$
t_0^+ < \tau_0 \ll t_0^- \,. \tag{3.5}
$$

FIG. 3. Plots of unperturbed (solid line), perturbed (dashed line) and partially compensated transmitted pulse envelopes for different values of ξ ; PBG structure contains a composite nonlinear materials in the layers with lower linear refractive index, $\varepsilon_1 = 0.2$.

In the following the term ''unperturbed pulse'' concerns the compression in the one-component PBG structure containing only the instantaneously responding self-focusing nonlinearity $t_0=0$; the "perturbed pulse" corresponds to the one-component PBG with $t_0 \neq 0$, the "compensated pulse" is applied to the two-component PBG compressor. The following consequences can be made from the analysis of the simulation results.

First of all, we could not find any evidence of the full compensation of soliton shape distortion for the short duration of the incident pulse, but observed a partial improvement of compression efficiency in comparison with the PBG crystal possessing the nonlinear response of single sign (Fig. 3). It has been found out that the transmitted pulse shape and compression efficiency depend primarily on ξ at the fixed parameters of a crystal, pulse duration and numerical constants of the fast responding nonlinearity. It implies that a more inertial defocusing material with the higher nonlinear coefficient provides the same result as a less inertial one but with lower n_2^- (Fig. 4). This is valid, if the strong inequality $\varepsilon_2 \ll 1$ is satisfied.

Unlike Ref. $[5]$, the maximum compression efficiency, not the optimal distortion compensation, is attained at much higher values of ξ , in particular when $\xi_{\text{max}}=0.2$. The further increase of parameter ξ , owing to either the increase of \tilde{n}_2 or the decrease of t_0^- , leads to the predominance of the defocusing mechanism over the self-focusing one $[Fig. 5(b)]$ and, as a consequence, to a pulse broadening $(Fig. 3)$. Also, the transmitted pulse acquires an asymmetrical shape with the steep leading and lengthy trailing edges—the well-known effect originating from the intensity-dependent group velocity in nonlinear media $[7]$. Moreover, the passage time through the crystal becomes shorter, since the light pulse envelope moves in the structure with the average nonlinear refractive index lowered [Fig. $5(b)$].

FIG. 4. Partially compensated transmitted pulse envelopes for different values of ε_2 and ξ . Compression in a composite PBG structure.

IV. LIGHT COMPRESSION UNDER AN INTERLEAVED ARRANGEMENT OF NONLINEARITIES IN PBG MATERIALS

The above consideration implicated that both the fastrelaxing and slow-relaxing nonlinearities are placed in the same layers of a gap structure and act simultaneously at every point of a medium. Although it is natural for a nonlinear material to possess several types of nonlinear mechanisms with different relaxation constants t_0 [9] or one can design a special mixture of matters to attain necessary properties of a composite material, it appears technologically easier to create a PBG crystal consisting of alternating layers of different nonlinearity types. In this case the contributions of the selffocusing and defocusing nonlinearities are separated in time and space, which, in turn, affects the soliton dynamics. Hence the question may be raised as to whether such a structural model of the photonic crystal can favor the soliton distortion compensation. In order to improve the PBG compressor efficiency, it is worth to fabricate a structure in which the fast-responding nonlinear material responsible for phase selfmodulation and compression is embedded in the layers of lower refractive index, n_2 in our geometry, while the slowresponding nonlinearity belongs to the layers with the higher refractive index n_1 . This choice of the structural configuration of a PBG crystal is connected to the well-known fact that at the blue edge's frequencies of the forbidden gap the electric field of radiation is primarily concentrated in the layers of lower n_0 (Fig. 6) due to the Bragg interference [10]. As a consequence, the nonlinear layers with n_2 play the dominant role in the nonlinear interaction. For this reason one should take greater values of the nonlinear coefficient \tilde{n}_2 in the layers with n_1 to attain the noticeable nonlinear response.

Figure 7 presents the calculated transmitted signal for the case of PBG compression where the nonlinearities occur intermittently. While comparing Figs. 7 and 3, one finds the

FIG. 5. Spatial distribution of the amplitude $|E|$ and nonlinear perturbation $\delta n/n_0$ inside a composite photonic crystal after 300 fs of interaction.

qualitative difference in the relaxation compensation dynamics corresponding to the various structural models of the PBG compressor.

First, there is no evidence of existence of the optimal value of compensation parameter ξ_{max} , and the compression efficiency monotonically rises with the growth of ξ . Secondly, we have found out that the peak intensity of transmitted signal at relatively high values of $\xi \sim 1-4$ can exceed the intensity of the unperturbed compressed pulse. Meanwhile, the regimes with $\xi > 1$ in the composite PBG crystals are characterized by the predominance of defocusing nonlinearity and the absence of compression. These peculiarities can be elucidated considering the spatial distribution of the induced nonlinear perturbation inside a crystal [Figs. 8 and Fig. $5(b)$. In the composite material the predominance of the defocusing response results in the full suppression of the

FIG. 6. Spatial distribution of the amplitude and nonlinear perturbation inside a photonic crystal with (a) composite nonlinear material, (b) alternating layers of different types of nonlinearity. Picture shot at 300 fs of interaction; overall crystal length is \sim 60 μ m.

influence of the fast-responding nonlinearity at every point of the crystal interior [Fig. $5(b)$] and, as a consequence, the phase self-modulation mechanism broadens the light pulse instead of compressing, since $\tilde{n}_2 k_2 > 0$. In the case of the intermittently arranged compensation scheme (Fig. 8) none of the nonlinear contributions of different types can be totally suppressed, since they are separated in time and space. Hence the principal formation mechanisms work by turns, in a periodic sequence of two stages: (1) the defocusing nonlinearity increases the sharpness of the leading pulse edge, owing to the intensity-dependent group velocity, (2) the selffocusing nonlinearity results in the fundamental compression, that narrows primarily the trailing pulse edge by means of phase self-modulation and GVD. Besides, the

FIG. 7. Plots of unperturbed (solid line), perturbed (dashed line) and compensated transmitted pulse envelopes for different values of ξ ; PBG structure with alternating nonlinear layers of different types, $\varepsilon_1=0.2$.

pulse experiences a certain acceleration due to the diminution of the average refractive index.

V. CONCLUSION

We have performed computer simulation of ultrashort pulse compression in the photonic crystals with the delayedresponse cubic nonlinearity. A technique is suggested that enables us, in principle, to suppress the destructive influence of the relaxation processes, for example, intropulse Raman scattering, on the Bragg soliton formation. The basic idea is that the soliton distortions caused by the relaxation of the fast self-focusing nonlinearity can be compensated by an additional defocusing nonlinearity having the much longer relaxation time. For the embodiment of this idea two conceivable structural models of PBG crystals are presented. That is, the first one is a composite-nonlinearity periodically stratified medium with the combined effect of two nonlinear components in each layer; the second one is a interleavednonlinearity PBG structure where the nonlinear components

FIG. 8. Spatial distribution of the amplitude $|E|$ and nonlinear perturbation $\delta n/n_0$ inside a photonic crystal with alternating nonlinear layers of different types; picture shot at 300 fs of interaction.

are distributed intermittently and act sequentially, layer-bylayer. Either model has its own merits.

It is relevant to make some remarks on the possible realization of two-component nonlinear photonic crystals. In our opinion, there are two promising directions in designing PBG compressors. The first one deals with the saturation of artificial or natural 3D photonic crystals with organic nonlinear liquids exhibiting picosecond recovery times of the thirdorder nonlinear susceptibility [9]. The currently existing technologies allow fabricating such multidimensional matrices on the base of synthetic opal crystals $[11]$. The second direction is the development of 1D semiconductor-dielectric PBG crystals utilizing narrow-gap semiconductors, providing a great variety of cubic nonlinear mechanisms of picosecond and subpicosecond range of relaxation times [12].

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